Effect of Moisture and Moisture Related Phenomena from Skylab

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Introduction

An attempt was made to find the effect of antenna pattern on radiometric measurements made by S-194 radiometer operating in L-band. The half-power beam width of S-194 sensor is 15°. The 15° solid cone centered about the vertical axis corresponds to the angular width of the beam between the half-power points of the pattern. The 15° angle will encompass a swath width of 60 nmi at the orbital altitude of 235 nmi. The first null of the major lobe of the antenna pattern is at 36° which encompasses a swath width of 145 nmi. About 97% of the energy is contained within the major lobe. The radiometric measurements by the sensor will be influenced to a greater extent by the brightness temperature of the material contained within the half-power beam width.

The radiant energy received by the antenna was sampled at such a rate as to ensure a minimum of 97% ground coverage overlap. The distance on the ground between centers of two consecutive resolution cells is about 2 nmi. Because of such a small distance, the difference in the radiometric antenna temperature measurement between two consectuive resolution cells is primarily due to random fluctuations due to noise rather than to any changes in the ground emission. Thus, for a meaningful analysis, it is important to establish the effect of antenna pattern to show at what distance the ground effects begin to predominate over those of noise. Before correlating the "ground truth" information with the antenna temperature, it is also important to establish the independence of radiometric measurements between the resolution cells at different center displacements.

The purpose of the analysis presented here is twofold. The first is to determine how the ground signal is modified by the antenna pattern and to establish the effect of noise. The second is to explore the possibility of reducing the effective resolution cell size by applying some kind of correction to the radiometric measurement. The possiblity of an improved estimate of the ground signal from the corrected radiometric measurements is explored.

The analysis is one-dimensional in the sense that only variations in the direction along the ground track are considered. In effect this means assuming a rectangular rather than circular antenna pattern; but, this assumption which greatly simplifies analysis, is not likely to give results significantly different from those with an assumed circular pattern.

Theory

Let $g(x)$ be the ground signal as a function of distance n, $a(x)$ the antenna pattern as a function of distance, $n(x)$ noise as a function of distance; then data, d(x) as a function of distance is given by:

$$d(x) = a(x) * g(x) + n(x) \tag{1}$$

Taking a Fourier transform of equation 1:

$$D(w) = A(w)\ G(w) + N(w) \tag{2}$$

If we wish to find an estimate $\hat{G}(w)$ of $G(w)$ by dividing in the frequency domain, we have:

$$\hat{G}(w) = \frac{D(w)}{A(w)} = G(w) + \frac{N(w)}{A(w)} \tag{3}$$

The simplified and approximate normalized antenna pattern for S-194 as given in the Sensor Performance Manual[1] is:

$$a(\Theta) = \exp(-158\sin^2\frac{\Theta}{2})$$

But for small angle Θ,

$$\sin\frac{\Theta}{2} \approx \frac{\sin\Theta}{2} \approx \frac{\tan\Theta}{2} = \frac{x}{2h}$$

where:

x = distance on the ground
h = altitude = 231 nmi

Thus:

$$a(x) = \exp(\frac{-x^2}{1350})$$

or:

$$a(x) = \exp(\frac{-x^2}{x_a{}^2}) \tag{4}$$

where $x_a{}^2 = 1350$, $x_a = 36.7$

Taking the Fourier transform of equation 4, we have:

$$A(w) = x_a\sqrt{\pi}\ e^{\frac{-w^2 x_a{}^2}{4}} \tag{5}$$

Assume for simplicity of analysis that the ground signal g(x) is also given by a Guassian form:

$$g(x) = g_0 \, e^{-\frac{x^2}{x_0^2}} \tag{6}$$

Thus, by taking the Fourier transform of Equation 6, we have:

$$G(w) = g_0 \, x_0 \sqrt{\pi} \, e^{\frac{-w^2 x_0^2}{4}} \tag{7}$$

We further describe the noise by a Gaussian form that decays between sampling points:

$$n(x) = n_0 \, e^{\frac{-x^2}{x_n^2}} \tag{8}$$

This implies that every time a sample is taken, a new noise of this size is introduced as given by Equation 8. The Fourier transform of Equation 8 is given by:

$$N(w) = n_0 x_n \sqrt{\pi} \, e^{\frac{-w^2 x_n^2}{4}} \tag{9}$$

By combining Equations 3,5,6 and 9, we have:

$$\hat{G}(w) = g_0 x_0 \sqrt{\pi} \, e^{\frac{-w^2 x_0^2}{4}} + n_0 \frac{x_n}{x_a} \, e^{\frac{w^2 (x_a^2 - x_n^2)}{4}} \tag{10}$$

To obtain $\hat{g}(x)$, the signal to be estimated, we have to take the inverse Fourier transform of Equation 10. The limits normally used on the inverse Fourier transform integral are $-\infty$ to $+\infty$. In the present case, the noise term, the one which has been multiplied by the inverse of the antenna pattern, is an ever increasing term with frequency and the inverse transform convergent. This implies that there will be more contribution to the noise from the high frequency components than from the small frequency components. These high frequency components are extremely small in the actual antenna pattern and they become important here because the smaller the number, the larger the value of its inverse.

It is clear that the factor multiplying the noise term will have to be truncated in the frequency domain. The problem is to establish at what point the truncation is necessary. If the noise term is truncated, it is clear that the signal term will have to be truncated accordingly.

The inverse Fourier transform of Equation 10 can be written as follows by assigning a finite limit on the integrals instead of ∞:

$$\hat{g}(x) = \frac{g_o x_o}{\sqrt{\pi}} \int_o^{\Omega} e^{\frac{-w^2 x_o^2}{4}} \cos xw dw + \frac{n_o x_n}{\pi x_a} \int_o^{\Omega} \cdot e^{\frac{w^2(x_a^2 - x_n^2)}{4}} \cos xw dw \qquad (11)$$

Truncating the noise term is equivalent to multiplying it by a rectangular frequency window function which is unity within the limits and zero outside. Such a window function is used for convenience here. Perhaps a window function with gradual cutoff would be somewhat superior to the rectangular one, but none were tried.

From Equation 2, we have:

$$D(w) = g_o x_a x_o \pi \, e^{\frac{-w^2(x_a^2 + x_o^2)}{4}} + \sqrt{\pi}\, n_o x_n \, e^{\frac{-w^2 x_n^2}{4}} \qquad (12)$$

Or, taking the inverse Fourier transform of Equation 12:

$$d(x) = \frac{g_o x_a x_o \sqrt{\pi}}{\sqrt{x_a^2 - x_o^2}} e^{\frac{-x^2}{x_o^2 + x_a^2}} + n_o e^{\frac{-x^2}{x_n^2}} \qquad (13)$$

As given in the Sensor Performance Report[1], the antenna temperature measured by the S-194 radiometer has uncertainty described by the standard deviation of the temperature measurement. For an average of 200°, the standard deviation is about 1°. Thus, we can assume a signal-to-noise ratio of 200 to 1 in the present analysis.

By assuming a signal-to-noise ratio of 200, from Equation 13, at $x=0$, we have:

$$\frac{g_o x_o x_a \sqrt{\pi}}{\sqrt{x_a^2+x_o^2}} = 200 n_o$$

Therefore:

$$n_o = \frac{g_o x_a x_o \sqrt{\pi}}{\sqrt{x_a^2+x_o^2}} \cdot \frac{1}{200} \qquad (14)$$

Also, Equation 11 can be rewritten as:

$$\hat{g}(x) = \frac{g_o x_o}{\sqrt{\pi}} \int_o^{\Omega} e^{-(w^2/w_1)^2} \cos xw dw +$$

$$\frac{n_o x_n}{x_a} \int_o^{\Omega} e^{(w/w_2)^2} \cos xw dw$$

where:

$$w_1 = 2/x_o, \text{ and } w_2 = 2/\sqrt{x_a^2-x_n^2} ,$$

we have:

$$\left(\frac{\Omega}{w_1}\right)^{w_1} = \left(\frac{\Omega}{w_2}\right)^{w_2} , \text{ Thus:}$$

$$\left(\frac{\Omega}{w_1}\right) = \left(\frac{\Omega}{w_2}\right) \frac{w_2}{w_1}$$

Let $\frac{\Omega}{w_2} = c$, Therefore:

$$\Omega_1 = \left(\frac{\Omega}{w_1}\right) w_1 = \left(\frac{\Omega}{w_2}\right) w_2 = c w_2 = \Omega_2$$

Thus:

$$\Omega = \Omega_1 = \Omega_2 = cw_2 \tag{15}$$

where c can be chosen arbirtarily. We have from Equation 4 that $x_a = 36.7$. Let $x_n = 1.0$ nmi.

We are assuming that samples are taken every two nmi. The Gaussian form of the noise is centered around the sampling point and it decays in between the sampling points. The value of 1.0 for x_n is then justified where the distance between the center of the successive resolution cells is two nmi.

To demonstrate the method, we further assume the following different values of x_o: $x_o = 5,10,20$ nmi. These different values are chosen because they will show how different signals are modified by the antenna pattern and to establish the trend of inverse antenna pattern modification. This will also demonstrate how well different types of signals can be reproduced.

By substituting the values of x_n, x_o and x_a in Equation 14, we have:

$$\begin{aligned} x_o &= 5, \; n_o = 0.042g_o \\ x_o &= 10, \; n_o = 0.082g_o \\ x_o &= 20, \; n_o = 0.149g_o \end{aligned} \tag{16}$$

Also, by choosing c arbitrarily, and evaluating the noise and signal integral numerically, we have ,at x=0, from Equation 11:

$x_o = 5$

C	Signal Integral	Noise Integral	Noise Term/ Signal Term
2.50	0.13102	6.22	60.5×10^{-4}
2.75	0.14301	20.44	186×10^{-4}
3.00	0.15469	77.31	600×10^{-4}
3.25	0.16604	335.30	2600×10^{-4}

$x_o = 10$

C	Signal Integral	Noise Integral	Noise Term/ Signal Term
2.50	0.11775	6.22	67.2×10^{-4}
2.75	0.12594	20.44	206×10^{-4}
3.00	0.13333	77.31	732×10^{-4}
3.25	0.13995	335.30	2940×10^{-4}

$x_o = 20$

C	Signal Integral	Noise Integral	Noise Term/ Signal Term
2.50	0.08393	6.22	85×10^{-4}
2.75	0.08572	20.44	274×10^{-4}
3.00	0.08692	77.31	1020×10^{-4}
3.25	0.08770	335.30	4390×10^{-4}

We only have to evaluate the noise integral at n=0 because we are assuming that a new, independent noise sample is centered on each data sampling point. The ratio between the noise term and the signal term was obtained from Equations 11, 15 and 16, and the values of the ingegrals given above. The limit, Ω, on the integrals corresponding to any value of c is given by:

$$\Omega = cw_2 = 2c/(x_a{}^2 - x_n{}^2)^{1/2}$$

For signal = 250, at x=0, we have from the above tables:

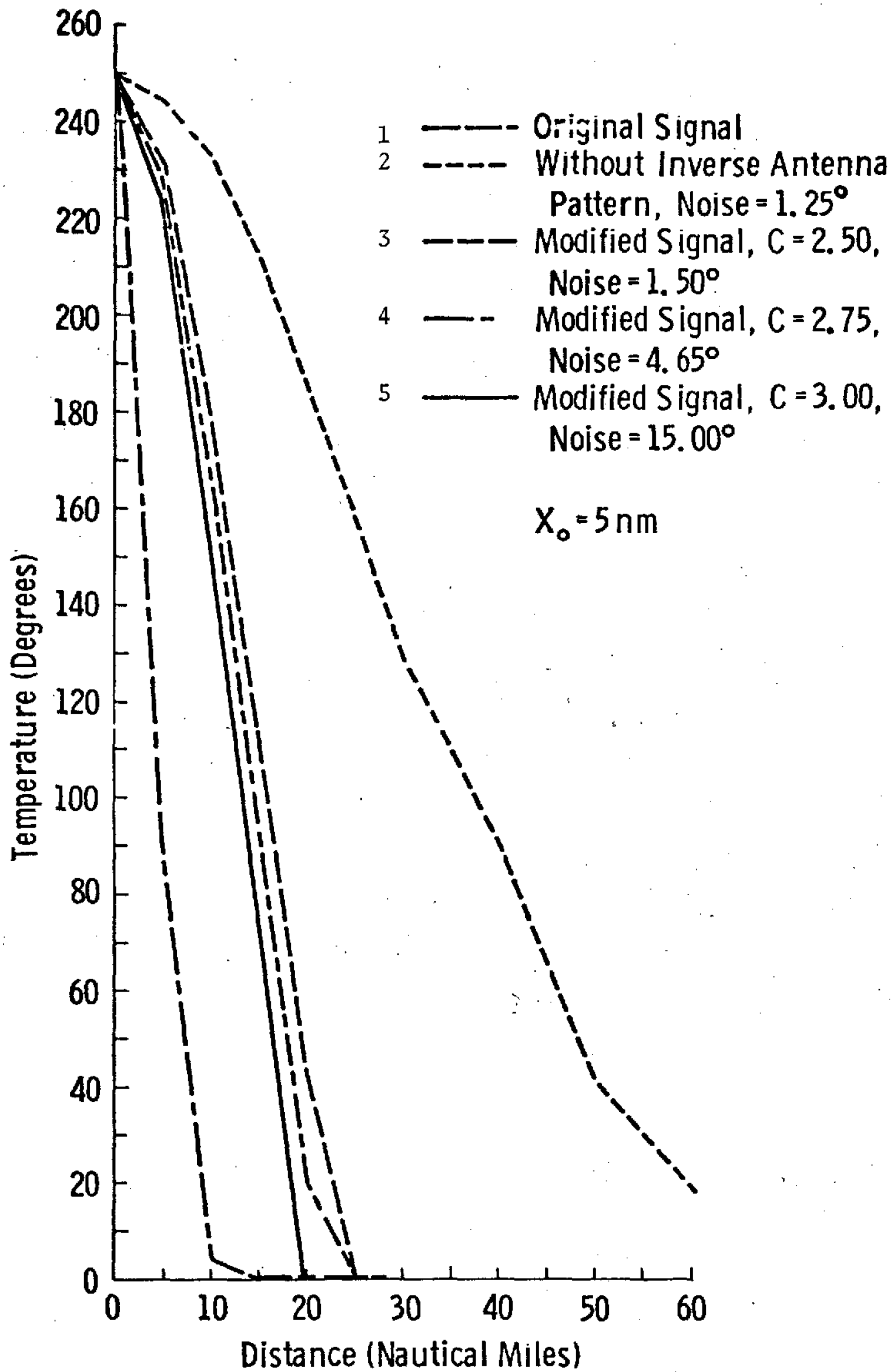
C	Ω	Signal	Noise		
			$x_o=5$	$x_o=10$	$x_o=20$
2.50	2.50/18.3	250.0	1.50	1.68	2.13
2.75	2.75/18.3	250.0	4.65	5.11	6.82
3.00	3.00/18.3	250.0	15.00	18.30	26.00
3.25	3.25/18.3	250.0	65.00	73.50	109.00

Thus, depending on where the data is chopped off in the frequency domain, the noise introduced is as given above and is statistically the same at every point. The signal in Equation 11 was evaluated for different n corresponding to different values of x_o (5, 10 and 20 nmi) and the results are given in Figures 1,2 and 3, respectively.

Each figure corresponding to different x_o shows the original signal given by Equation 6, the modified signal given by Equation 11 for different limits and the signal without inverse antenna pattern given by Euqation 13.

Discussion of Results and Conclusions

In the attached figures corresponding to different x_o, the original signal which had to be reproduced is given by curve 1. If no inverse antenna pattern was used, this signal would look like curve 2. In this case, the original signal had been modified by the antenna pattern.



1 Original Signal
2 Without Inverse Antenna Pattern, Noise = 1.25°
3 Modified Signal, C = 2.50, Noise = 1.50°
4 Modified Signal, C = 2.75, Noise = 4.65°
5 Modified Signal, C = 3.00, Noise = 15.00°
X₀ = 5 nm
Temperature (Degrees)
Distance (Nautical Miles)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
10
30
50

Figure 1

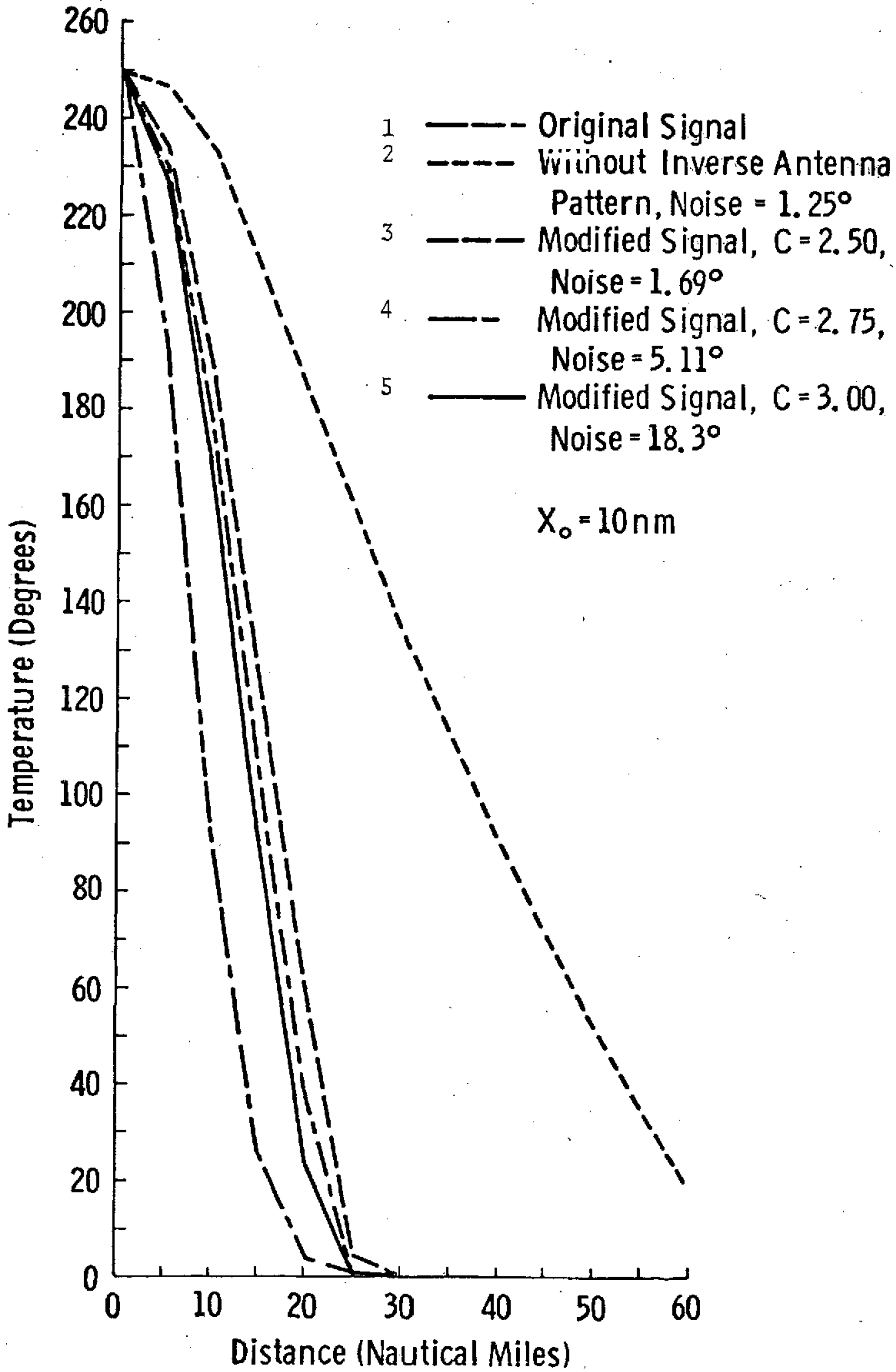

260
240
220
200
180
160
140
120
100
80
60
40
20
0
Temperature (Degrees)
0
10
20
30
40
50
60
Distance (Nautical Miles)
1 Original Signal
2 Without Inverse Antenna Pattern, Noise = 1.25°
3 Modified Signal, C = 2.50, Noise = 1.69°
4 Modified Signal, C = 2.75, Noise = 5.11°
5 Modified Signal, C = 3.00, Noise = 18.3°
X₀ = 10 nm

Figure 2

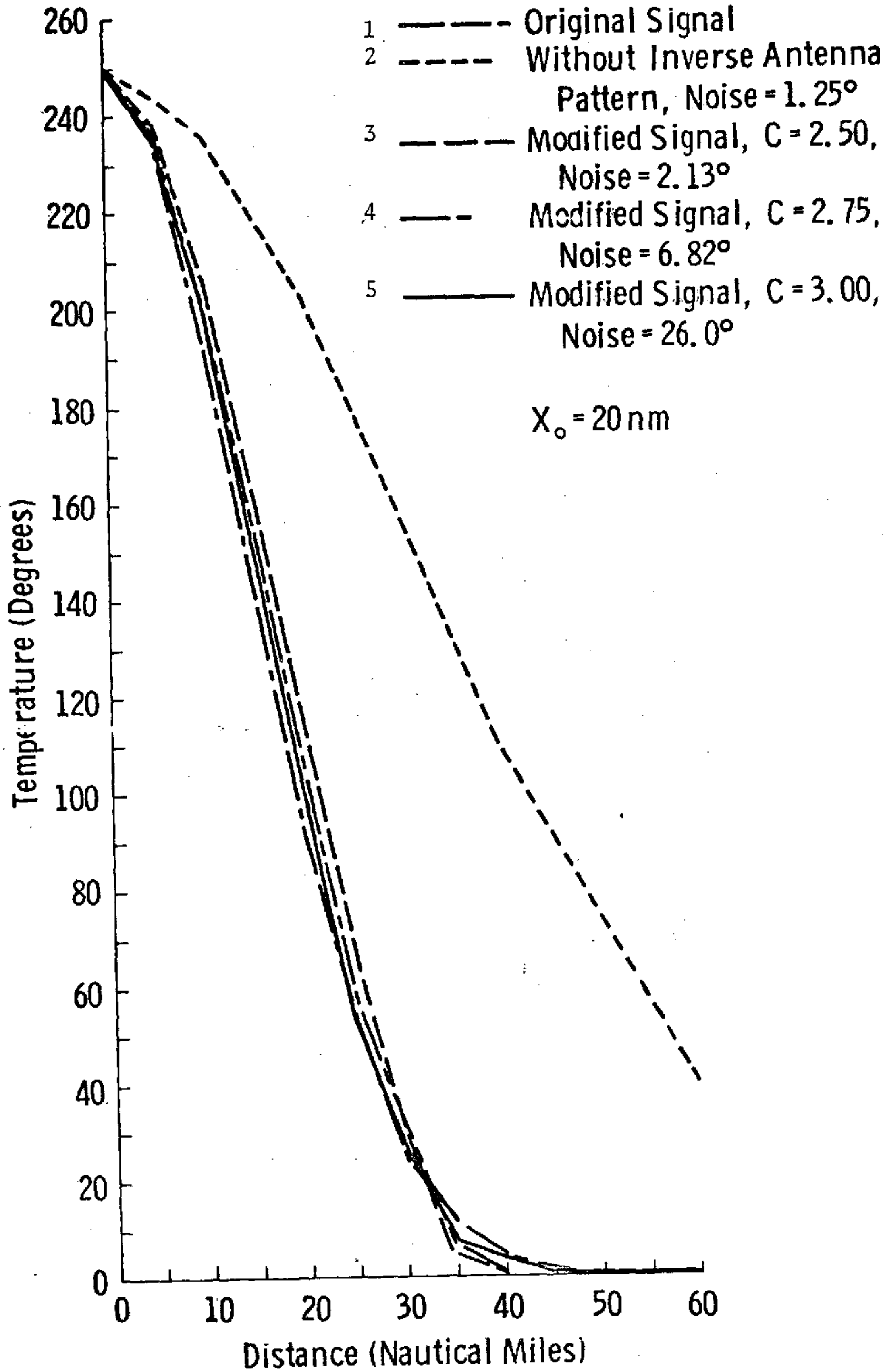


Figure 3

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If we decide to correct or modify the signal by utilizing the inverse antenna pattern, we get curves 3 to 5 for different limits. As the limit in the frequency domain increases, the mean of the modified signal approaches the original signal, but the uncertainty or the noise contribution increases significantly. This means that we could reproduce the original signal exactly in the ensemble mean by choosing the limit to be ∞ in the absence of noise, but the uncertainty of an individual experiment due to noise will be so high that it will be impractical to do so. For example, when the limit on the noise integral corresponds to $c=3.0$, the modified signal as given by curve 5 is almost the same as the original signal for $x_0=20$, but it is different for $x_0=5$. The uncertainty in the estimation increases from 65.0 for $x_0=5$ to 109.0 for $x_0=20$. This implies that the uncertainty in the estimated signal also depends on the original signal.

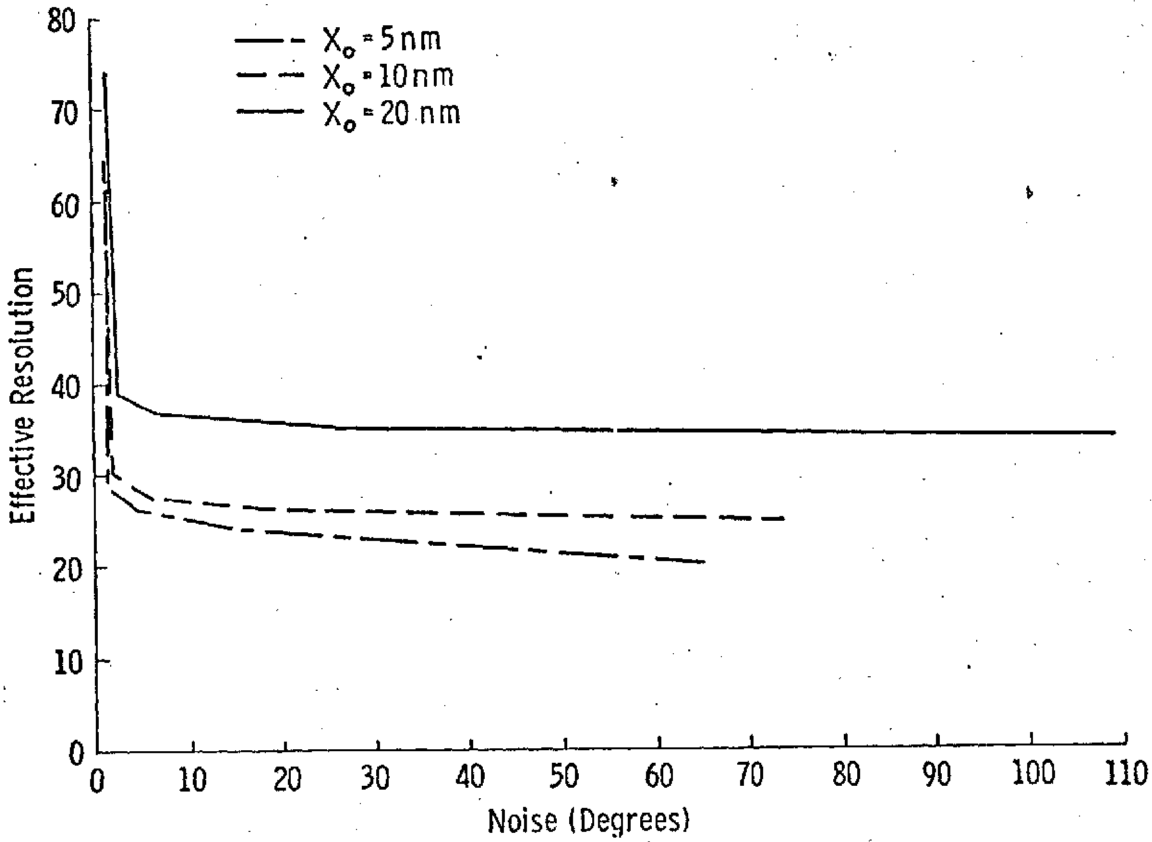
The question to be decided is that in what range of values the uncertainty in the corrected or modified signal is practical to have and can be tolerated. If we can tolerate an uncertainty of about $\pm 5°$, then the limit chosen should correspond to $c \approx 2.75$. With this much uncertainty, the modified or corrected signal as given by curve 4 is not close to the original for $x_0=5$; but, as the x_0 increases the estimated signal gets closer and closer to the original signal until it is almost the same. Even for the worst case of estimation for $x_0=5$, the modified signal is considerably better than the unmodified or uncorrected signal given by curve 2.

If the limit corresponding to c=2.50 is chosen, then the uncertainty in the signal will be about 2.0°, but the estimated signal for $x_0=5$ will be worse than that for c=2.75.

The reduction in the size of the resolution cell because of modification of the signal utilizing the inverse pattern is evident from the following:

	x_0		
	5	10	20
Half-power width of unmodified signal.	31.25	32.00	37.00
Half-power width of modified signal, c=2.75.	13.00	13.75	18.00
Improvement in the resolution cell size by a factor of:	2.4	2.33	2.05

Thus, the size of the resolution cell is reduced from about 70 nmi in the unmodified signal case to about 35 nmi in the case of the modified signal. A figure showing the variation of noise with the effective resolution cell size for different values of x_0 is attached (Fig. 4). The sharpness in the transition of noise is to be noted. A very small decrease in the effective resolution cell size produces a great increase in noise.



X_o = 5 nm
X_o = 10 nm
X_o = 20 nm
Effective Resolution
Noise (Degrees)
0
10
20
30
40
50
60
70
80
90
100
110

Figure 4

How to Obtain the Modified Signal

Let d(x) be the signal as a function of distance which represents the antenna temperature measured by the S-194 sensor. Let a(x) be the antenna pattern given by the following:

$$a(x) = e^{-x^2/x_a^2} \tag{17}$$

where $x_a^2 = 1350$. The Fourier transform of Equation 17 is given by:

$$A(w) = x_a\sqrt{\pi}\, e^{\frac{-w^2 x_a^2}{4}}$$

$$\frac{1}{A(w)} = A^{-1}(w) = \frac{1}{x_a\sqrt{\pi}}\, e^{\frac{+w^2 x_a^2}{4}} \tag{18}$$

Take Fourier transform of d(x) and let it be denoted by D(w), then $\hat{G}(w)$ is given by:

$$\hat{G}(w) = \frac{D(w)}{x_a\sqrt{\pi}}\, e^{\frac{w^2 x_a^2}{4}} \tag{19}$$

$\hat{g}(x)$ can be obtained by the following equation:

$$\hat{g}(n) = \frac{2}{2\pi x_a\sqrt{\pi}} \int_0^{w_1} D(w)\, e^{\frac{w^2 x_a^2}{4}} \cos xwdw$$

where $w_1 = c/18.3$ or, for c= 2.75:

$$\hat{g}(x) = \frac{1}{\pi^{(3/2)}\, x_a} \int_0^{w_1} D(w)\, e^{\frac{w^2 x_a^2}{4}} \cos xwdw \tag{20}$$

The modified signal as given by Equation 20 can then be used instead of the original data.